NAG Toolbox for MATLAB

g08da

1 Purpose

g08da calculates Kendall's coefficient of concordance on k independent rankings of n objects or individuals.

2 Syntax

```
[w, p, ifail] = g08da(x, k, 'n', n)
```

3 Description

Kendall's coefficient of concordance measures the degree of agreement between k comparisons of n objects, the scores in the ith comparison being denoted by

$$x_{i1}, x_{i2}, \ldots, x_{in}.$$

The hypothesis under test, H_0, often called the null hypothesis, is that there is no agreement between the comparisons, and this is to be tested against the alternative hypothesis, H_1, that there is some agreement.

The n scores for each comparison are ranked, the rank r_{ij} denoting the rank of object j in comparison i, and all ranks lying between 1 and n. Average ranks are assigned to tied scores.

For each of the n objects, the k ranks are totalled, giving rank sums R_j, for $j = 1, 2, \ldots, n$. Under H_0, all the R_j would be approximately equal to the average rank sum $k(n+1)/2$. The total squared deviation of the R_j from this average value is therefore a measure of the departure from H_0 exhibited by the data. If there were complete agreement between the comparisons, the rank sums R_j would have the values $k, 2k, \ldots, nk$ (or some permutation thereof). The total squared deviation of these values is $k^2(n^3 - n)/12$.

Kendall's coefficient of concordance is the ratio

$$W = \frac{\sum_{j=1}^{n} \left(R_j - \frac{1}{2}k(n+1)\right)^2}{\frac{1}{12}k^2(n^3 - n)}$$

and lies between 0 and 1, the value 0 indicating complete disagreement, and 1 indicating complete agreement.

If there are tied rankings within comparisons, W is corrected by subtracting $k\sum T$ from the denominator, where $T = \sum(t^3 - t)/12$, each t being the number of occurrences of each tied rank within a comparison, and the summation of T being over all comparisons containing ties.

g08da returns the value of W, and also an approximation, p, of the significance of the observed W. (For $n > 7, k(n-1)W$ approximately follows a χ^2_{n-1} distribution, so large values of W imply rejection of H_0.) H_0 is rejected by a test of chosen size α if $p < \alpha$. If $n \leq 7$, tables should be used to establish the significance of W (e.g., Table R of Siegel 1956).

4 References

Siegel S 1956 *Non-parametric Statistics for the Behavioral Sciences* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **x(ldx,n) – double array**

ldx, the first dimension of the array, must be at least **k**.

$\mathbf{x}(i,j)$ must be set to the value x_{ij} of object j in comparison i, for $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,n$.

2: **k – int32 scalar**

k, the number of comparisons.

Constraint: $\mathbf{k} \geq 2$.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **x**.

n, the number of objects.

Constraint: $\mathbf{n} \geq 2$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldx, rnk

5.4 Output Parameters

1: **w – double scalar**

The value of Kendall's coefficient of concordance, W.

2: **p – double scalar**

The approximate significance, p, of W.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 2$.

ifail $= 2$

On entry, $\mathbf{ldx} < \mathbf{k}$.

ifail $= 3$

On entry, $\mathbf{k} \leq 1$.

7 Accuracy

All computations are believed to be stable. The statistic W should be accurate enough for all practical uses.

8 Further Comments

The time taken by g08da is approximately proportional to the product nk.

9 Example

```
x = [1, 4.5, 2, 4.5, 3, 7.5, 6, 9, 7.5, 10;
     2.5, 1, 2.5, 4.5, 4.5, 8, 9, 6.5, 10, 6.5;
     2, 1, 4.5, 4.5, 4.5, 4.5, 8, 8, 8, 10];
k = int32(3);
[w, p, ifail] = g08da(x, k)
```

```
w =
    0.8277
p =
    0.0078
ifail =
           0
```